§ 7.1. ТЕОРЕТИЧЕСКИЕ ВОПРОСЫ

1) Определения двойного и тройного интегралов. Их геометрический и физический смысл.

2) Основные свойства двойных и тройных интегралов.

3) Теорема о среднем для двойного и тройного интегралов.

4) Вычисление двойных интегралов двумя последовательными интегрированиями (случай прямоугольной области).

5) Вычисление двойных интегралов двумя последовательными интегрированиями (общий случай).

6) Замена переменных в двойном интеграле.

7) Якобиан, его геометрический смысл.

8) Двойной интеграл в полярных координатах.

9) Тройной интеграл в цилиндрических координатах.

10) Тройной интеграл в сферических координатах.

§ 7.2. ТЕОРЕТИЧЕСКИЕ УПРАЖНЕНИЯ

1) Пользуясь определением двойного интеграла, доказать, что

$$\iint\limits_{x^2+y^2\leq R^2} x^m y^n dxdy = 0,$$

если m и n — натуральные числа и, по меньшей мере, одно из них нечетно.

2) С помощью теоремы о среднем найти

$$\lim_{R\to 0}\frac{1}{\pi R^2}\iint\limits_{x^2+y^2\leq R^2} f(x,y)dxdy,$$

где $f(x,y)$ — непрерывная функция.

'3) Оценить интеграл

$$\iiint\frac{dxdydz}{\sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2}}, x_0^2+y_0^2+z_0^2 > R^2,$$

т. е. указать, между какими значениями заключена его величина.

4) Вычислить двойной интеграл $\iint_D f(x,y)dxdy$, если область D — прямоугольник $\{a\leq x\leq b, c\leq y\leq d\}$, а $f(x,y) = F''_{xy}(x,y)$.

5) Доказать равенство

$$\iint_D f(x)g(y)dxdy = \int\limits_a^b f(x)dx\int\limits_c^d g(y)dy,$$

если область D —прямоугольник $\{a\leq x\leq b, c\leq y\leq d\}$.

6) Доказать формулу Дирихле.

$$\int\limits_0^a dx \int\limits_0^x f(x,y)dy = \int\limits_0^a dy \int\limits_y^a f(x,y)dx, a > 0.$$

7) Пользуясь формулой Дирихле, доказать равенство

$$\int\limits_0^a dy \int\limits_0^y f(x)dx = \int\limits_0^a (a-x)f(x)dx.$$

8) Какой из интегралов больше

$$\int\limits_0^1 dx \int\limits_0^1 dy \int\limits_0^1 f(x,y,z)dz \text{ или } \int\limits_0^1 dx \int\limits_0^{1-x} dy \int\limits_0^{1-x-y} f(x,y,z)dz,$$

если $f(x,y,z) > 0$?

Задача 1. Изменить порядок интегрирования.

1. $\int\limits_{-2}^{-1} dy \int\limits_{-\sqrt{2+y}}^0 fdx + \int\limits_{-1}^0 dy \int\limits_{-\sqrt{-y}}^0 fdx.$

9. $\int\limits_{-\sqrt2}^{-1} dx \int\limits_0^{\sqrt{2-x^2}} fdy + \int\limits_{-1}^0 dx \int\limits_0^{x^2} fdy.$

2. $\int\limits_0^1 dy \int\limits_{-\sqrt{y}}^0 fdx + \int\limits_1^{\sqrt2} dy \int\limits_{-\sqrt{2-y^2}}^0 fdx.$

10. $\int\limits_{-2}^{-\sqrt3} dx \int\limits_{-\sqrt{4-x^2}}^0 fdy + \int\limits_{-\sqrt3}^0 dx \int\limits_{\sqrt{4-x^2}-2}^0 fdy.$

3. $\int\limits_0^1 dy \int\limits_0^y fdx + \int\limits_1^{\sqrt2} dy \int\limits_{\sqrt{2-y}}^{\sqrt{2-y^2}} fdx.$

11. $\int\limits_0^1 dx \int\limits_{1-x^2}^1 fdy + \int\limits_1^e dx \int\limits_{\ln x}^1 fdy.$

4. $\int\limits_0^1 dy \int\limits_0^{\sqrt{y}} fdx + \int\limits_1^2 dy \int\limits_0^{\sqrt{2-y}} fdx.$

12. $\int\limits_0^1 dy \int\limits_0^{\sqrt[3]{y}} fdx + \int\limits_1^2 dy \int\limits_0^{2-y} fdx.$

5. $\int\limits_{-\sqrt2}^{-1} dy \int\limits_{-\sqrt{2-x^2}}^0 fdx + \int\limits_{-1}^0 dy \int\limits_x^0 fdx.$

13. $\int\limits_0^{\pi.4} dy \int\limits_0^{\sin y} fdx + \int\limits_{\pi/4}^{\pi/2} dy \int\limits_0^{\cos y} fdx.$

6. $\int\limits_0^{1/\sqrt2} dy \int\limits_0^{\arcsin y} fdx + \int\limits_{1/\sqrt2}^1 dy \int\limits_0^{\arccos y} fdx.$

14. $\int\limits_{-2}^{-1} dx \int\limits_{-(2+x)}^0 fdy + \int\limits_{-1}^0 dx \int\limits_{\sqrt[3]{x}}^0 fdy.$

7. $\int\limits_{-2}^{-1} dy \int\limits_0^{\sqrt{2+y}} fdx + \int\limits_{-1}^0 dy \int\limits_0^{\sqrt{-y}} fdx.$

15. $\int\limits_0^{\pi/4} dy \int\limits_0^{\sin y} fdx + \int\limits_{\pi/4}^{\pi/2} dy \int\limits_0^{\cos y} fdx.$

8. $\int\limits_0^1 dy \int\limits_{-\sqrt{y}}^0 fdx + \int\limits_1^e dy \int\limits_{-1}^{-\ln y} fdx.$

16. $\int\limits_0^1 dy \int\limits_{-\sqrt{y}}^0 fdx + \int\limits_1^2 dy \int\limits_{-\sqrt{2-y}}^0 fdx.$

17. $\int\limits_{0}^{1}dy\int\limits_{-y}^{0}fdx+\int\limits_{1}^{\sqrt{2}}dy\int\limits_{-\sqrt{2-y^2}}^{0}fdx.$

18. $\int\limits_{0}^{1}dy\int\limits_{0}^{y^3}fdx+\int\limits_{1}^{2}dy\int\limits_{0}^{2-y}fdx.$

19. $\int\limits_{0}^{\sqrt{3}}dx\int\limits_{\sqrt{4-x^2}-2}^{0}fdy+\int\limits_{\sqrt{3}}^{2}dx\int\limits_{-\sqrt{4-x^2}}^{0}fdy.$

20. $\int\limits_{-2}^{-1}dy\int\limits_{-(2+y)}^{0}fdx+\int\limits_{-1}^{0}dy\int\limits_{\sqrt[3]{y}}^{0}fdx.$

21. $\int\limits_{0}^{1}dy\int\limits_{-0}^{y}fdx+\int\limits_{1}^{e}dy\int\limits_{\ln y}^{1}fdx.$

22. $\int\limits_{0}^{1}dx\int\limits_{0}^{x^2}fdy+\int\limits_{1}^{\sqrt{2}}dx\int\limits_{0}^{\sqrt{2-x^2}}fdy.$

23. $\int\limits_{0}^{\pi/4}dx\int\limits_{0}^{\sin x}fdy+\int\limits_{\pi/4}^{\pi/2}dx\int\limits_{0}^{\cos x}fdy.$

24. $\int\limits_{-\sqrt{2}}^{1}dy\int\limits_{-\sqrt{2-y^2}}^{0}fdx+\int\limits_{-1}^{0}dy\int\limits_{y}^{0}fdx.$

25. $\int\limits_{0}^{1}dx\int\limits_{0}^{x^3}fdy+\int\limits_{1}^{2}dx\int\limits_{0}^{2-x}fdy.$

26. $\int\limits_{0}^{\sqrt{3}}dx\int\limits_{0}^{2-\sqrt{4-x^2}}fdy+\int\limits_{\sqrt{3}}^{2}dx\int\limits_{0}^{\sqrt{4-x^2}}fdy.$

27. $\int\limits_{0}^{1}dx\int\limits_{-\sqrt{x}}^{0}fdy+\int\limits_{1}^{2}dx\int\limits_{-\sqrt{2-x}}^{0}fdy.$

28. $\int\limits_{0}^{1}dx\int\limits_{0}^{x}fdy+\int\limits_{1}^{\sqrt{2}}dx\int\limits_{0}^{\sqrt{2-x^2}}fdy.$

29. $\int\limits_{0}^{1}dy\int\limits_{0}^{\sqrt{y}}fdx+\int\limits_{1}^{\sqrt{2}}dy\int\limits_{0}^{\sqrt{2-y^2}}fdx.$

30. $\int\limits_{0}^{1}dx\int\limits_{0}^{\sqrt{x}}fdy+\int\limits_{1}^{2}dx\int\limits_{0}^{\sqrt{2-x}}fdy.$

31. $\int\limits_{-2}^{-\sqrt{3}}dx\int\limits_{0}^{\sqrt{4-x^2}}fdy+\int\limits_{-\sqrt{3}}^{0}dx\int\limits_{0}^{2-\sqrt{4-x^2}}fdy.$

Задача 2. Вычислить.

1. $\iint\limits_{D}(12x^2y^2+16x^3y^3)dxdy; D: x=1, y=x^2, y=-\sqrt{x}.$

2. $\iint\limits_{D}(9x^2y^2+48x^3y^3)dxdy; D: x=1, y=\sqrt{x}, y=-x^2.$

3. $\iint\limits_{D}(36x^2y^2-96x^3y^3)dxdy; D: x=1, y=\sqrt[3]{x}, y=-x^3.$

4. $\iint\limits_{D}(18x^2y^2+32x^3y^3)dxdy; D: x=1, y=x^3, y=-\sqrt[3]{x}.$

5. $\iint\limits_{D}(27x^2y^2+48x^3y^3)dxdy; D: x=1, y=x^2, y=-\sqrt[3]{x}(x\geq0).$

6. $\iint\limits_{D}(18x^2y^2+32x^3y^3)dxdy; D: x=1, y=\sqrt[3]{x}, y=-x^2(x\geq0).$

7. $\iint\limits_{D}(18x^2y^2+32x^3y^3)dxdy; D: x=1, y=x^3, y=-\sqrt{x}.$

8. $\iint\limits_{D}(27x^2y^2+48x^3y^3)dxdy; D: x=1, y=\sqrt{x}, y=-x^3.$

9. $\iint\limits_{D}(4xy+16x^2y^2)dxdy; D: x=1, y=x^2, y=-\sqrt{x}.$

10. $\iint\limits_{D}(12xy+9x^2y^2)dxdy; D: x=1, y=\sqrt{x}, y=-x^2.$

11. $\iint\limits_{D}(8xy+9x^2y^2)dxdy; D: x=1, y=\sqrt[3]{x}, y=-x^3.$

12. $\iint\limits_{D}(24xy+18x^2y^2)dxdy; D: x=1, y=x^3, y=-\sqrt[3]{x}.$

13. $\iint\limits_{D}(12xy+27x^2y^2)dxdy; D: x=1, y=x^2, y=-\sqrt[3]{x}(x\ge 0).$

14. $\iint\limits_{D}(8xy+18x^2y^2)dxdy; D: x=1, y=\sqrt[3]{x}, y=-x^2(x\ge 0).$

15. $\iint\limits_{D}\left(\frac{4}{5}xy+\frac{9}{11}x^2y^2\right)dxdy; D: x=1, y=x^3, y=-\sqrt{x}.$

16. $\iint\limits_{D}\left(\frac{4}{5}xy+9x^2y^2\right)dxdy; D: x=1, y=\sqrt{x}, y=-x^3.$

17. $\iint\limits_{D}(24xy+48x^3y^3)dxdy; D: x=1, y=x^2, y=-\sqrt{x}.$

18. $\iint\limits_{D}(6xy+24x^3y^3)dxdy; D: x=1, y=\sqrt{x}, y=-x^2.$

19. $\iint\limits_{D}(4xy+16x^3y^3)dxdy; D: x=1, y=\sqrt[3]{x}, y=-x^3.$

20. $\iint\limits_{D}(4xy+16x^3y^3)dxdy; D: x=1, y=x^3, y=-\sqrt[3]{x}.$

21. $\iint\limits_{D}(44xy+16x^3y^3)dxdy; D: x=1, y=x^2, y=-\sqrt[3]{x}(x\ge 0).$

22. $\iint\limits_{D}(4xy+176x^3y^3)dxdy; D: x=1, y=\sqrt[3]{x}, y=-x^3(x\ge 0).$

23. $\iint\limits_{D}(xy-4x^3y^3)dxdy; D: x=1, y=x^3, y=-\sqrt{x}.$

24. $\iint\limits_D (4xy + 176x^3 y^3)dxdy; D: x = 1, y = \sqrt{x}, y = -x^3.$

25. $\iint\limits_D \left(6x^2 y^2 + \frac{25}{3} x^3 y^3\right)dxdy; D: x = 1, y = x^2, y = -\sqrt{x}.$

26. $\iint\limits_D (9x^2 y^2 + 25x^4 y^4)dxdy; D: x = 1, y = \sqrt{x}, y = -x^2.$

27. $\iint\limits_D \left(3x^2 y^2 + \frac{50}{3} x^4 y^4\right)dxdy; D: x = 1, y = \sqrt[3]{x}, y = -x^3.$

28. $\iint\limits_D (9x^2 y^2 + 25x^4 y^4)dxdy; D: x = 1, y = x^3, y = -\sqrt[3]{x}.$

29. $\iint\limits_D (54x^2 y^2 + 150x^4 y^4)dxdy; D: x = 1, y = x^2, y = -\sqrt[3]{x}(x \ge 0).$

30. $\iint\limits_D (xy - 9x^5 y^5)dxdy; D: x = 1, y = \sqrt[3]{x}, y = -x^2(x \ge 0).$

31. $\iint\limits_D (54x^2 y^2 + 150x^3 y^3)dxdy; D: x = 1, y = x^3, y = -\sqrt{x}.$

Задача 3. Вычислить.

1. $\iint\limits_D ye^{xy/2}dxdy; D: y = \ln 2, y = \ln 3, x = 2, x = 4.$

2. $\iint\limits_D y^2 \sin\frac{xy}{2} dy; D: x = 0, y = \sqrt{\pi}, y = \frac{x}{2}.$

3. $\iint\limits_D y\cos xydxdy; D: y = \frac{\pi}{2}, y = \pi, x = 1, x = 2.$

4. $\iint\limits_D y^2 e^{-xy/4}dxdy; D: x = 0, y = 2, y = x.$

5. $\iint\limits_D y\sin xydxdy; D: y = \frac{\pi}{2}, y = \pi, x = 1, x = 2.$

6. $\iint\limits_D y^2 \cos\frac{xy}{2} dxdy; D: x = 0, y = \sqrt{\frac{\pi}{2}}, y = \frac{x}{2}.$

7. $\iint\limits_D 4ye^{2xy}dxdy; D: y = \ln 3, y = \ln 4, x = \frac{1}{2}, x = 1.$

8. $\iint\limits_{D} 4y^2 \sin xy dx dy; D: x = 0, y = \sqrt{\dfrac{\pi}{2}}, y = x.$

9. $\iint\limits_{D} y \cos 2x dx dy; D: y = \dfrac{\pi}{2}, y = \pi, x = \dfrac{1}{2}, x = 1.$

10. $\iint\limits_{D} y^2 e^{-xy/8} dx dy; D: x = 0, y = 2, y = \dfrac{x}{2}.$

11. $\iint\limits_{D} 12 y \sin 2xy dx dy; D: y = \dfrac{\pi}{4}, y = \dfrac{\pi}{2}, x = 2, x = 3.$

12. $\iint\limits_{D} y^2 \cos xy dx dy; D: x = 0, y = \sqrt{\pi}, y = x.$

13. $\iint\limits_{D} y e^{xy/4} dx dy; D: y = \ln 2, y = \ln 3, x = 4, x = 8.$

14. $\iint\limits_{D} 4y^2 \sin 2xy dx dy; D: x = 0, y = \sqrt{2\pi}, y = 2x.$

15. $\iint\limits_{D} 2y \cos 2xy dx dy; D: y = \dfrac{\pi}{4}, y = \dfrac{\pi}{2}, x = 1, x = 2.$

16. $\iint\limits_{D} y^2 e^{-xy/2} dx dy; D: x = 0, y = \sqrt{2}, y = x.$

17. $\iint\limits_{D} y \sin xy dx dy; D: y = \pi, y = 2\pi, x = \dfrac{1}{2}, x = 1.$

18. $\iint\limits_{D} y^2 \cos 2xy dx dy; D: x = 0, y = \sqrt{\dfrac{\pi}{2}}, y = \dfrac{x}{2}.$

19. $\iint\limits_{D} 8 e^{4xy} dx dy; D: y = \ln 3, y = \ln 4, x = \dfrac{1}{4}, x = \dfrac{1}{2}.$

20. $\iint\limits_{D} 3y^2 \sin \dfrac{xy}{2} dx dy; D: x = 0, y = \sqrt{\dfrac{4\pi}{3}}, y = \dfrac{2}{3}x.$

21. $\iint\limits_{D} y \cos xy dx dy; D: y = \pi, y = 3\pi, x = \dfrac{1}{2}, x = 1.$

22. $\iint\limits_{D} y^2 e^{-xy/2} dx dy; D: x = 0, y = 1, y = \dfrac{x}{2}.$

23. $\iint\limits_{D} y \sin 2xy dx dy; D: y = \dfrac{\pi}{2}, y = \dfrac{3\pi}{2}, x = \dfrac{1}{2}, x = 3.$

24. $\iint\limits_{D} y^2 \cos xy \, dxdy; D: x = 0, y = \sqrt{\pi}, y = 2x.$

25. $\iint\limits_{D} 6ye^{xy/3} dxdy; D: y = \ln 2, y = \ln 3, x = 3, x = 6.$

26. $\iint\limits_{D} y^2 \sin \frac{xy}{2} dxdy; D: x = 0, y = \sqrt{\pi}, y = x.$

27. $\iint\limits_{D} y \cos 2x \, dxdy; D: y = \frac{\pi}{2}, y = \frac{3\pi}{2}, x = \frac{1}{2}, x = 2.$

28. $\iint\limits_{D} y^2 e^{-xy/8} dxdy; D: x = 0, y = 4, y = 2x.$

29. $\iint\limits_{D} 3y \sin xy \, dxdy; D: y = \frac{\pi}{2}, y = 3\pi, x = 1, x = 3.$

30. $\iint\limits_{D} y^2 \cos \frac{xy}{2} dxdy; D: x = 0, y = \sqrt{2\pi}, y = 2x.$

31. $\iint\limits_{D} 12ye^{6xy} dxdy; D: y = \ln 3, y = \ln 4, x = \frac{1}{6}, x = \frac{1}{3}.$

Задача 4. Вычислить.

1. $\iiint\limits_{V} 2y^2 e^{xy} dxdydz; V \begin{cases} x = 0, y = 1, y = x, \\ z = 0, z = 1. \end{cases}$

2. $\iiint\limits_{V} x^2 z \sin(xyz) dxdydz; V \begin{cases} x = 2, y = \pi, z = 1, \\ x = 0, y = 0, z = 0. \end{cases}$

3. $\iiint\limits_{V} y^2 ch(2xy) dxdydz; V \begin{cases} x = 0, y = -2, y = 4x, \\ z = 0, z = 2. \end{cases}$

4. $\iiint\limits_{V} 8y^2 z e^{2xyz} dxdydz; V \begin{cases} x = -1, y = 2, z = 1, \\ x = 0, y = 0, z = 0. \end{cases}$

5. $\iiint\limits_{V} x^2 sh(3xy) dxdydz; V \begin{cases} x = 1, y = 2x, y = 0, \\ z = 0, z = 36. \end{cases}$

6. $\iiint\limits_{V} y^2 z \cos xyz \, dxdydz; V \begin{cases} x = 1, y = \pi, z = 2, \\ x = 0, y = 0, z = 0. \end{cases}$

7. $\iiint\limits_{V} y^2 \cos\left(\dfrac{\pi}{4} xy\right) dxdydz; V\begin{cases} x = 0, y = -1, y = \dfrac{x}{2}, \\ z = 0, z = -\pi^2. \end{cases}$

8. $\iiint\limits_{V} x^2 z \sin\dfrac{xyz}{4} dxdydz; V\begin{cases} x = 1, y = 2\pi, z = 4, \\ x = 0, y = 0, z = 0. \end{cases}$

9. $\iiint\limits_{V} y^2 e^{-xy} dxdydz; V\begin{cases} x = 0, y = -2, y = 4x, \\ z = 0, z = 1. \end{cases}$

10. $\iiint\limits_{V} 2y^2 z e^{xyz} dxdydz; V\begin{cases} x = 1, y = 1, z = 1, \\ x = 0, y = 0, z = 0. \end{cases}$

11. $\iiint\limits_{V} y^2 ch(2xy) dxdydz; V\begin{cases} x = 0, y = 1, y = x, \\ z = 0, z = 8. \end{cases}$

12. $\iiint\limits_{V} x^2 z sh(xyz) dxdydz; V\begin{cases} x = 2, y = 1, z = 1, \\ x = 0, y = 0, z = 0. \end{cases}$

13. $\iiint\limits_{V} y^2 e^{xy/2} dxdydz; V\begin{cases} x = 0, y = 2, y = 2x, \\ z = 0, z = -1. \end{cases}$

14. $\iiint\limits_{V} y^2 z \cos\dfrac{xyz}{3} dxdydz; V\begin{cases} x = 3, y = 1, z = 2\pi, \\ x = 0, y = 0, z = 0. \end{cases}$

15. $\iiint\limits_{V} y^2 \cos\left(\dfrac{\pi xy}{2}\right) dxdydz; V\begin{cases} x = 0, y = -1, y = x, \\ z = 0, z = 2\pi^2. \end{cases}$

16. $\iiint\limits_{V} 2x^2 z sh(xyz) dxdydz; V\begin{cases} x = 1, y = -1, z = 1, \\ x = 0, y = 0, z = 0. \end{cases}$

17. $\iiint\limits_{V} y^2 \cos(\pi xy) dxdydz; V\begin{cases} x = 0, y = 1, y = 2x, \\ z = 0, z = .\pi^2 \end{cases}$

18. $\iiint\limits_{V} 2x^2 z sh(2xyz) dxdydz; V\begin{cases} x = 2, y = \dfrac{1}{2}, z = \dfrac{1}{2}, \\ x = 0, y = 0, z = 0. \end{cases}$

19. $\iiint\limits_{V} x^2 sh(2xy) dxdydz; V\begin{cases} x = -1, y = x, y = 0, \\ z = 0, z = 8. \end{cases}$

20. $\iiint\limits_{V} x^2 z \sin\dfrac{xyz}{2} dxdydz; V\begin{cases} x = 1, y = 4, z = \pi, \\ x = 0, y = 0, z = 0. \end{cases}$

21. $\iiint\limits_V y^2 ch(xy) dxdydz; V \begin{cases} x=0, y=-1, y=x, \\ \qquad z=0, z=2. \end{cases}$

22. $\iiint\limits_V y^2 zch(xyz) dxdydz; V \begin{cases} x=1, y=1, z=1, \\ x=0, y=0, z=0. \end{cases}$

23. $\iiint\limits_V x^2 \sin\left(\dfrac{\pi}{2} xy\right) dxdydz; V \begin{cases} x=2, y=x, y=0, \\ \qquad z=0, z=\pi. \end{cases}$

24. $\iiint\limits_V y^2 z\cos\dfrac{xyz}{9} dxdydz; V \begin{cases} x=9, y=1, z=2\pi, \\ x=0, y=0, z=0. \end{cases}$

25. $\iiint\limits_V x^2 \sin(\pi xy) dxdydz; V: x=1, y=2x, y=0, z=0, z=4\pi.$

26. $\iiint\limits_V y^2 zch\left(\dfrac{xyz}{2}\right) dxdydz; V: x=2, y=-1, z=2, x=0, y=0, z=0.$

27. $\iiint\limits_V y^2 ch(3xy) dxdydz; V: x=0, y=2, y=6x, z=0, z=-3.$

28. $\iiint\limits_V 2y^2 zch(2xyz) dxdydz; V: x=\dfrac{1}{2}, y=2, z=-1, x=0, y=0, z=0.$

29. $\iiint\limits_V x^2 \sin(4\pi xy) dxdydz; V: x=1, y=\dfrac{x}{2}, y=0, z=0, z=8\pi.$

30. $\iiint\limits_V 8y^2 ze^{-xyz} dxdydz; V: x=2, y=-1, z=2, x=0, y=0, z=0.$

31. $\iiint\limits_V x^2 sh(xy) dxdydz; V: x=2, y=\dfrac{x}{2}, y=0, z=0, z=1.$

Задача 5. Вычислить.

1. $\iiint\limits_V x dxdydz; V: y=10x, y=0, x=1, z=xy, z=0.$

2. $\iiint\limits_V \dfrac{dxdydz}{\left(1+\dfrac{x}{3}+\dfrac{y}{4}+\dfrac{z}{8}\right)^4}; V: \dfrac{x}{3}+\dfrac{y}{4}+\dfrac{z}{8}=1, x=0, y=0, z=0.$

3. $\iiint\limits_V 15(y^2+z^2) dxdydz; V: z=x+y, x+y=1, x=0, y-0, z=0.$

4. $\iiint\limits_V (3x+4y)dxdydz; V: y=x, y=0, x=1, z=5(x^2+y^2).$

5. $\iiint\limits_V (1+2x^3)dxdydz; V: y=9x, y=0, x=1, z=\sqrt{xy}, z=0.$

6. $\iiint\limits_V (27+54y^3)dxdydz; V: y=x, y=0, x=1, z=\sqrt{xy}, z=0.$

7. $\iiint\limits_V ydxdydz; V: y=15x, y=0, x=1, z=xy, z=0.$

8. $\iiint\limits_V \dfrac{dxdydz}{\left(1+\dfrac{x}{16}+\dfrac{y}{8}+\dfrac{z}{3}\right)^5}; V: \dfrac{x}{16}+\dfrac{y}{8}+\dfrac{z}{3}=1, x=0, y=0, z=0.$

9. $\iiint\limits_V (3x^2+y^2)dxdydz; V: z=10y, x+y=1, x=0, y=0, z=0.$

10. $\iiint\limits_V (15x+30z)dxdydz; V: z=x^2+3y^2, z=0, y=x, y=0, x=1.$

11. $\iiint\limits_V (4+8z^3)dxdydz; V: y=x, y=0, x=1, z=\sqrt{xy}, z=0.$

12. $\iiint\limits_V (1+2x^3)dxdydz; V: y=36x, y=0, x=1, z=\sqrt{xy}, z=0.$

13. $\iiint\limits_V 21xzdxdydz; V: y=x, y=0, x=2, z=xy, z=0.$

14. $\iiint\limits_V \dfrac{dxdydz}{\left(1+\dfrac{x}{10}+\dfrac{y}{8}+\dfrac{z}{3}\right)^6}; V: \dfrac{x}{10}+\dfrac{y}{8}+\dfrac{z}{3}=1, x=0, y=0, z=0.$

15. $\iiint\limits_V (x^2+3y^2)dxdydz; V: z=10x, x+y=1, x=0, y=0, z=0.$

16. $\iiint\limits_V (60y+90z)dxdydz; V: y=x, y=0, x=1, z=x^2+y^2, z=0.$

17. $\iiint\limits_V \left(\dfrac{10}{3}x+\dfrac{5}{3}\right)dxdydz; V: y=9x, y=0, x=1, z=\sqrt{xy}, z=0.$

18. $\iiint\limits_V (9+18z)dxdydz; V: y=4x, y=0, x=1, z=\sqrt{xy}, z=0.$

19. $\iiint\limits_V 3y^2dxdydz; V: y=2x, y=0, x=2, z=xy, z=0.$

20. $\iiint\limits_V \dfrac{dxdydz}{\left(1 + \dfrac{x}{2} + \dfrac{y}{4} + \dfrac{z}{6}\right)^4}; V: \dfrac{x}{2} + \dfrac{y}{4} + \dfrac{z}{6} = 1, x = 0, y = 0, z = 0.$

21. $\iiint\limits_V x^2 dxdydz; V: z = 10(x + 3y), x + y = 1, x = 0, y = 0, z = 0.$

22. $\iiint\limits_V (8y + 12z)dxdydz; V: y = x, y = 0, x = 1, z = 3x^2 + 2y^2, z = 0.$

23. $\iiint\limits_V 63(1 + 2\sqrt{y})dxdydz; V: y = x, y = 0, x = 1, z = \sqrt{xy}, z = 0.$

24. $\iiint\limits_V (x + y)dxdydz; V: y = x, y = 0, x = 1, z = 30x^2 + 60y^2, z = 0.$

25. $\iiint\limits_V \dfrac{dxdydz}{\left(1 + \dfrac{x}{6} + \dfrac{y}{4} + \dfrac{z}{16}\right)^5}; V: \dfrac{x}{6} + \dfrac{y}{4} + \dfrac{z}{16} = 1, x = 0, y = 0, z = 0.$

26. $\iiint\limits_V xyz\,dxdydz; V: y = x, y = 0, x = 2, z = xy, z = 0.$

27. $\iiint\limits_V y^2 dxdydz; V: z = 10(3x + y), x + y = 1, x = 0, y = 0, z = 0.$

28. $\iiint\limits_V \left(5x + \dfrac{3z}{2}\right)dxdydz; V: y = x, y = 0, x = 1, z = x^2 + 15y^2, z = 0.$

29. $\iiint\limits_V (x^2 + 4y^2)dxdydz; V: z = 20(2x + y), x + y = 1,, x = 0, y = 0, z = 0.$

30. $\iiint\limits_V \dfrac{dxdydz}{\left(1 + \dfrac{x}{8} + \dfrac{y}{3} + \dfrac{z}{5}\right)^6}; V: \dfrac{x}{8} + \dfrac{y}{3} + \dfrac{z}{5} = 1, x = 0, y = 0, z = 0.$

31. $\iiint\limits_V x^2 z\,dxdydz; V: y = 3x, y = 0, x = 2, z = xy, z = 0.$

Задача 6. Найти площадь фигуры, ограниченной данными линиями.

1. $y = \dfrac{3}{x}, y = 4e^x, y = 3, y = 4.$

2. $x = \sqrt{36 - y^2}, x = 6 - \sqrt{36 - y^2}.$

3. $x^2 + y^2 = 72, 6y = -x^2 \, (y \le 0).$

4. $x = 8 - y^2, x = -2y.$

5. $y = \dfrac{3}{x}, y = 8e^x, y = 3, y = 8.$

6. $y = \dfrac{\sqrt{x}}{2}, y = \dfrac{1}{2x}, x = 16.$

7. $x = 5 - y^2, x = -4y.$

8. $x^2 + y^2 = 12, -\sqrt{6}y = x^2 \, (y \le 0).$

9. $y = \sqrt{12 - x^2}, y = 2\sqrt{3} - \sqrt{12 - x^2}, x = 0, (x \ge 0).$

10. $y = \dfrac{3}{2}\sqrt{x}, y = \dfrac{3}{2x}, x = 9.$

11. $y = \sqrt{24 - x^2}, 2\sqrt{3}y = x^2, x = 0, (x \ge 0).$

12. $y = \sin x, y = \cos x, x = 0 \, (x \ge 0).$

13. $y = 20 - x^2, y = -8x.$

14. $y = \sqrt{18 - x^2}, y = 3\sqrt{2} - \sqrt{18 - x^2}.$

15. $y = 32 - x^2, y = -4x.$

16. $y = \dfrac{2}{x}, y = 5e^x, y = 2, y = 5.$

17. $x^2 + y^2 = 36, 3\sqrt{2}y = x^2 \, (y \ge 0).$

18. $y = 3\sqrt{x}, y = \dfrac{3}{x}, x = 4.$

19. $y = 6 - \sqrt{36 - x^2}, y = \sqrt{36 - x^2}, x = 0, (x \ge 0).$

20. $y = \dfrac{25}{4} - x^2, y = x - \dfrac{5}{2}.$

21. $y = \sqrt{x}, y = \dfrac{1}{x}, x = 16.$

22. $y = \dfrac{2}{x}, y = 7e^x, y = 2, y = 7.$

23. $y = 27 - y^2, x = -6y.$

24. $x = \sqrt{72 - y^2}, 6x = y^2, y = 0(y \geq 0).$

25. $y = \sqrt{6 - x^2}, y = \sqrt{6} - \sqrt{6 - x^2}.$

26. $y = \dfrac{3}{2}\sqrt{x}, y = \dfrac{3}{2x}, x = 4.$

27. $y = \sin x, y = \cos x, x = 0 (x \leq 0.$

28. $y = \dfrac{1}{x}, y = 6e^x, y = 1, y = 6.$

29. $y = 3\sqrt{x}, y = \dfrac{3}{x}, x = 94.$

30. $y = 11 - x^2, y = -10x.$

31. $x^2 + y^2 = 12, x\sqrt{6} = y^2 (x \geq 0).$

Задача 7. Найти площадь фигуры, ограниченной данными линиями.

1. $y^2 - 2y + x^2 = 0, y^2 - 4y + x^2 = 0, y = \dfrac{x}{\sqrt{3}}, y = \sqrt{3}x.$

2. $x^2 - 4x + y^2 = 0, x^2 - 8x + y^2 = 0, y = 0, y = \dfrac{x}{\sqrt{3}}.$

3. $y^2 - 6y + x^2 = 0, y^2 - 8y + x^2 = 0, y = \dfrac{x}{\sqrt{3}}, y = \sqrt{3}x.$

4. $x^2 - 2x + y^2 = 0, x^2 - 4x + y^2 = 0, y = 0, y = x.$

5. $y^2 - 8y + x^2 = 0, y^2 - 10y + x^2 = 0, y = \dfrac{x}{\sqrt{3}}, y = \sqrt{3}x.$

6. $x^2 - 4x + y^2 = 0, x^2 - 8x + y^2 = 0, y = 0, y = x.$

7. $y^2 - 4y + x^2 = 0, y^2 - 6y + x^2 = 0, y = x, x = 0.$

8. $x^2 - 2x + y^2 = 0, x^2 - 10x + y^2 = 0, y = 0, y = \sqrt{3}x.$

9. $y^2 - 6y + x^2 = 0, y^2 - 10y + x^2 = 0, y = x, x = 0.$

10. $x^2 - 2x + y^2 = 0, x^2 - 4x + y^2 = 0, y = \dfrac{x}{\sqrt{3}}, y = \sqrt{3}x.$

11. $y^2 - 2y + x^2 = 0, y^2 - 4y + x^2 = 0, y = \sqrt{3}x, x = 0.$

12. $x^2 - 2x + y^2 = 0, x^2 - 6x + y^2 = 0, y = \dfrac{x}{\sqrt{3}}, y = \sqrt{3}x.$

13. $y^2 - 4y + x^2 = 0, y^2 - 6y + x^2 = 0, y = \sqrt{3}x, x = 0.$

14. $x^2 - 2x + y^2 = 0, x^2 - 8x + y^2 = 0, y = \dfrac{x}{\sqrt{3}}, y = \sqrt{3}x.$

15. $y^2 - 2y + x^2 = 0, y^2 - 6y + x^2 = 0, y = \dfrac{x}{\sqrt{3}}, x = 0.$

16. $x^2 - 2x + y^2 = 0, x^2 - 4x + y^2 = 0, y = 0, y = \dfrac{x}{\sqrt{3}}.$

17. $y^2 - 2y + x^2 = 0, y^2 - 10y + x^2 = 0, y = \dfrac{x}{\sqrt{3}}, y = \sqrt{3}x.$

18. $x^2 - 2x + y^2 = 0, x^2 - 6x + y^2 = 0, y = 0, y = \dfrac{x}{\sqrt{3}}.$

19. $y^2 - 4y + x^2 = 0, y^2 - 10y + x^2 = 0, y = \dfrac{x}{\sqrt{3}}, y = \sqrt{3}x.$

20. $x^2 - 2x + y^2 = 0, x^2 - 6x + y^2 = 0, y = 0, y = x.$

21. $y^2 - 2y + x^2 = 0, y^2 - 4y + x^2 = 0, y = x, x = 0.$

22. $x^2 - 2x + y^2 = 0, x^2 - 4x + y^2 = 0, y = 0, y = \sqrt{3}x.$

23. $y^2 - 6y + x^2 = 0, y^2 - 8y + x^2 = 0, y = x, x = 0.$

24. $x^2 - 4x + y^2 = 0, x^2 - 8x + y^2 = 0, y = 0, y = \sqrt{3}x.$

25. $y^2 - 4y + x^2 = 0, y^2 - 8y + x^2 = 0, y = x, x = 0.$

26. $x^2 - 4x + y^2 = 0, x^2 - 8x + y^2 = 0, y = \dfrac{x}{\sqrt{3}}, y = \sqrt{3}x.$

27. $y^2 - 4y + x^2 = 0, y^2 - 8y + x^2 = 0, y = \sqrt{3}x, x = 0.$

28. $x^2 - 4x + y^2 = 0, x^2 - 6x + y^2 = 0, y = \dfrac{x}{\sqrt{3}}, y = \sqrt{3}x.$

29. $y^2 - 2y + x^2 = 0, y^2 - 10y + x^2 = 0, y = \dfrac{x}{\sqrt{3}}, x = 0.$

30. $x^2 - 6x + y^2 = 0, x^2 - 10x + y^2 = 0, y = \dfrac{x}{\sqrt{3}}, y = \sqrt{3}x.$

31. $y^2 - 4y + x^2 = 0, y^2 - 8y + x^2 = 0, y = \dfrac{x}{\sqrt{3}}, x = 0.$

Задача 8. Пластинка D задана ограничивающими ее кривыми, μ-поверхностная плотность. Найти массу пластинки.

1. $D: x = 1, y = 0, y^2 = 4x (y \geq 0); \mu = 7x^2 + y.$

2. $D: x^2 + y^2 = 1, x^2 + y^2 = 4, x = 0, y = 0 (x \geq 0, y \geq 0); \mu = \dfrac{x + y}{x^2 + y^2}.$

3. $D: x = 1, y = 0, y^2 = 4x (y \geq 0); \mu = \dfrac{7x^2}{2} + 5y.$

4. $D: x^2 + y^2 = 9, x^2 + y^2 = 16, x = 0, y = 0 (x \geq 0, y \geq 0); \mu = \dfrac{2x + 5y}{x^2 + y^2}.$

5. $D: x = 2, y = 0, y^2 = 2x (y \geq 0); \mu = \dfrac{7x^2}{8} + 2y.$

6. $D: x^2 + y^2 = 1, x^2 + y^2 = 16, x = 0, y = 0 (x \geq 0, y \geq 0); \mu = \dfrac{x + y}{x^2 + y^2}.$

7. $D: x = 2, y = 0, y^2 = \dfrac{x}{2} (y \geq 0); \mu = \dfrac{7x^2}{2} + 6y.$

8. $D: x^2 + y^2 = 4, x^2 + y^2 = 25, x = 0, y = 0 (x \geq 0, y \leq 0); \mu = \dfrac{2x - 3y}{x^2 + y^2}.$

9. $D: x = 1, y = 0, y^2 = 4x (y \geq 0); \mu = x + 3y^2.$

10. $D: x^2 + y^2 = 1, x^2 + y^2 = 9, x = 0, y = 0 (x \geq 0, y \leq 0); \mu = \dfrac{x - y}{x^2 + y^2}.$

11. $D: x = 1, y = 0, y^2 = x (y \geq 0); \mu = 3x + 6y^2.$

12. $D: x^2 + y^2 = 9, x^2 + y^2 = 25, x = 0, y = 0 (x \leq 0, y \geq 0); \mu = \dfrac{2y - x}{x^2 + y^2}.$

13. $D: x = 2, y = 0, y^2 = \dfrac{x}{2} (y \geq 0); \mu = 2x + 3y^2.$

14. $D: x^2 + y^2 = 9, x^2 + y^2 = 16, x = 0, y = 0 (x \geq 0, y \geq 0); \mu = \dfrac{2x + 5y}{x^2 + y^2}.$

15. $D : x = \dfrac{1}{2}, y = 0, y^2 = 8x (y \geq 0); \mu = 7x + 3y^2.$

16. $D : x^2 + y^2 = 9, x^2 + y^2 = 16, x = 0, y = 0 (x \leq 0, y \geq 0); \mu = \dfrac{2y - 5x}{x^2 + y^2}.$

17. $D : x = 1, y = 0, y^2 = 4x (y \geq 0); \mu = 7x^2 + 2y.$

18. $D : x^2 + y^2 = 1, x^2 + y^2 = 16, x = 0, y = 0 (x \geq 0, y \geq 0); \mu = \dfrac{x + 3y}{x^2 + y^2}.$

19. $D : x = 2, y^2 = 4x, y = 0 (y \geq 0); \mu = \dfrac{7x^2}{4} + \dfrac{y}{2}.$

20. $D : x^2 + y^2 = 1, x^2 + y^2 = 4, x = 0, y = 0 (x \geq 0, y \geq 0); \mu = \dfrac{x + 2y}{x^2 + y^2}.$

21. $D : x = 2, y = 0, y^2 = 2x (y \geq 0); \mu = \dfrac{7x^2}{4} + y.$

22. $D : x^2 + y^2 = 1, x^2 + y^2 = 9, x = 0, y = 0 (x \geq 0, y \leq 0); \mu = \dfrac{2x - y}{x^2 + y^2}.$

23. $D : x = 2, y = 0, y^2 = \dfrac{x}{2} (y \geq 0); \mu = \dfrac{7x^2}{2} + 8y.$

24. $D : x^2 + y^2 = 1, x^2 + y^2 = 25, x = 0, y = 0 (x \geq 0, y \leq 0); \mu = \dfrac{x - 4y}{x^2 + y^2}.$

25. $D : x = 1, y = 0, y^2 = 4x (y \geq 0); \mu = 6x + 3y^2.$

26. $D : x^2 + y^2 = 4, x^2 + y^2 = 16, x = 0, y = 0 (x \geq 0, y \leq 0); \mu = \dfrac{3x - y}{x^2 + y^2}.$

27. $D : x = 2, y = 0, y^2 = \dfrac{x}{2} (y \geq 0); \mu = 4x + 6y^2.$

28. $D : x^2 + y^2 = 4, x^2 + y^2 = 9, x = 0, y = 0 (x \leq 0, y \geq 0); \mu = \dfrac{y - 4x}{x^2 + y^2}.$

29. $D : x = \dfrac{1}{2}, y = 0, y^2 = 2x (y \geq 0); \mu = 4x + 9y^2.$

30. $D : x^2 + y^2 = 4, x^2 + y^2 = 9, x = 0, y = 0 (x \leq 0, y \geq 0); \mu = \dfrac{y - 2x}{x^2 + y^2}.$

31. $D: x = \dfrac{1}{4}, y = 0, y^2 = 16x (y \geq 0); \mu = 16x + \dfrac{9y^2}{2}.$

Задача 9. Пластинка D задана ограничивающими ее кривыми, μ -поверхностная плотность. Найти массу пластинки.

1. $D: x^2 + \dfrac{y^2}{4} \leq 1; \mu = y^2.$

15. $D: \dfrac{x^2}{4} + y^2 \leq 1, x \geq 0, y \geq 0; \mu = 30x^3 y^7.$

2. $D: 1 \leq \dfrac{x^2}{9} + \dfrac{y^2}{4} \leq 2; y \geq 0; y \leq \dfrac{2}{3}x; \mu = \dfrac{y}{x}$

16. $D: 1 \leq \dfrac{x^2}{9} + \dfrac{y^2}{4} \leq 3; y \geq 0; y \leq \dfrac{2}{3}x; \mu = \dfrac{y}{x}.$

3. $D: \dfrac{x^2}{9} + \dfrac{y^2}{25} \leq 1, y \geq 0; \mu = x^2 y.$

17. $D: x^2 + \dfrac{y^2}{25} \leq 1, y \geq 0; \mu = 7x^4 y.$

4. $D: \dfrac{x^2}{9} + \dfrac{y^2}{25} \leq 1, y \geq 0; \mu = \dfrac{7x^2 y}{18}.$

18. $D: x^2 + \dfrac{y^2}{9} \leq 1, y \geq 0; \mu = 35x^4 y^3.$

5. $D: 1 \leq \dfrac{x^2}{4} + y^2 \leq 4; y \geq 0; y \leq \dfrac{x}{2}; \mu = \dfrac{8y}{x^3}.$

19. $D: \dfrac{x^2}{4} + \dfrac{y^2}{9} \leq 1; \mu = x^2.$

6. $D: \dfrac{x^2}{9} + y^2 \leq 1, x \geq 0; \mu = 7xy^6.$

20. $D: 1 \leq x^2 + \dfrac{y^2}{16} \leq 9; y \geq 0; y \leq 4x; \mu = \dfrac{y}{x^3}.$

7. $D: \dfrac{x^2}{4} + y^2 \leq 1; \mu = 4y^4.$

21. $D: \dfrac{x^2}{9} + y^2 \leq 1, x \geq 0; \mu = 11xy^8.$

8. $D: 1 \leq \dfrac{x^2}{4} + \dfrac{y^2}{9} \leq 4; x \geq 0; y \geq \dfrac{3x}{2}x; \mu = \dfrac{x}{y}.$

22. $D: 1 \leq \dfrac{x^2}{4} + \dfrac{y^2}{16} \leq 5; x \geq 0; y \geq 2x; \mu = \dfrac{x}{y}.$

9. $D: 1 \leq \dfrac{x^2}{16} + \dfrac{y^2}{4} \leq 4; y \geq 0; y \geq \dfrac{x}{2}; \mu = \dfrac{x}{y}.$

23. $D: 1 \leq \dfrac{x^2}{9} + \dfrac{y^2}{16} \leq 5; x \geq 0; y \geq \dfrac{2x}{3}; \mu = \dfrac{x}{y}.$

10. $D: \dfrac{x^2}{4} + \dfrac{y^2}{9} \leq 1, x \geq 0, y \geq 0; \mu = x^3 y.$

24. $D: \dfrac{x^2}{4} + \dfrac{y^2}{9} \leq 1, x \geq 0, y \geq 0; \mu = x^5 y.$

11. $D: \dfrac{x^2}{4} + y^2 \leq 1, x \geq 0, y \geq 0; \mu = 6x^3 y^3.$

25. $D: \dfrac{x^2}{4} + \dfrac{y^2}{25} \leq 1; \mu = x^4.$

12. $D: 1 \leq \dfrac{x^2}{4} + y^2 \leq 25; x \geq 0; y \geq \dfrac{x}{2}; \mu = \dfrac{x}{y^3}.$

26. $D: x^2 + \dfrac{y^2}{4} \leq 1, x \geq 0, y \geq 0; \mu = 15x^5 y^3.$

13. $D: \dfrac{x^2}{9} + \dfrac{y^2}{4} \leq 1; \mu = x^2 y^2.$

27. $D: 1 \leq \dfrac{x^2}{4} + \dfrac{y^2}{9} \leq 36; x \geq 0; y \geq \dfrac{2}{3}x; \mu = \dfrac{9x}{y^3}.$

14. $D: \dfrac{x^2}{16} + y^2 \leq 1; x \geq 0, y \geq 0; \mu = 5xy^7.$

28. $D: \dfrac{x^2}{100} + y^2 \leq 1, x \geq 0, y \geq 0; \mu = 6xy^9.$

29. $D: \dfrac{x^2}{16} + y^2 \leq 1, x \geq 0, y \geq 0; \mu = 105\, x^3 y^9.$

31. $D: 1 \leq \dfrac{x^2}{16} + y^2 \leq 3; x \geq 0; y \geq \dfrac{x}{4}; \mu = \dfrac{x}{y^5}.$

30. $D: 1 \leq \dfrac{x^2}{16} + y^2 \leq 3; x \geq 0; y \geq \dfrac{4}{3}x; \mu = \dfrac{27\,y}{x^5}.$

Задача 10. Найти объем тела, заданного ограничивающими его поверхностями.

1. $y = 16\sqrt{2x}, y = \sqrt{2x}, z = 0, x + z = 2.$

2. $y = 5\sqrt{x}, y = \dfrac{5x}{3}, z = 0, z = 5 + \dfrac{5\sqrt{x}}{3}.$

3. $x^2 + y^2 = 2, y = \sqrt{x}, y = 0, z = 0, z = 15x.$

4. $x + y = 2, y = \sqrt{x}, z = 12\,y, z = 0.$

5. $x = 20\sqrt{2y}, x = 5\sqrt{2y}, z = 0, x + y = \dfrac{1}{2}.$

6. $x = \dfrac{5\sqrt{y}}{2}, x = \dfrac{5y}{6}, z = 0, z = \dfrac{5}{6}(3 + \sqrt{y}).$

7. $x^2 + y^2 = 2, x = \sqrt{y}, x = 0, z = 0, z = 30y.$

8. $x + y = 2, x = \sqrt{y}, z = \dfrac{12x}{5}, z = 0.$

9. $y = 17\sqrt{2x}, y = 2\sqrt{2x}, z = 0, x + z = \dfrac{1}{2}.$

10. $y = \dfrac{5\sqrt{x}}{3}, y = \dfrac{5x}{9}, z = 0, z = \dfrac{5(3 + \sqrt{x})}{9}.$

11. $x^2 + y^2 = 8, y = \sqrt{2x}, y = 0, z = 0, z = \dfrac{15x}{11}.$

12. $x + y = 4, y = \sqrt{2x}, z = 3\,y, z = 0.$

13. $x = \dfrac{5\sqrt{y}}{6}, x = \dfrac{5y}{18}, z = 0, z = \dfrac{5}{18}(3 + \sqrt{y}).$

14. $x = 19\sqrt{2y}, x = 4\sqrt{2y}, z = 0, z + y = 2.$

15. $x^2 + y^2 = 8, x = \sqrt{2y}, x = 0, z = \dfrac{30y}{11}, z = 0.$

16. $x + y = 4, x = \sqrt{2y}, z = \dfrac{3x}{5}, z = 0.$

17. $y = 6\sqrt{3x}, y = \sqrt{3x}, z = 0, x + z = 3.$

18. $y = \dfrac{5}{6}\sqrt{x}, y = \dfrac{5}{18}x, z = 0, z = \dfrac{5}{18}(3 + \sqrt{x}).$

19. $x^2 + y^2 = 18, y = \sqrt{3x}, y = 0, z = 0, z = \dfrac{5x}{11}.$

20. $x + y = 6, y = \sqrt{3x}, z = 4\,y, z = 0.$

21. $x = 7\sqrt{3y}, x = 2\sqrt{3y}, z = 0, z + y = 3.$

22. $y = \dfrac{5\sqrt{y}}{3}, x = \dfrac{5y}{9}, z = 0, z = \dfrac{5(3 + \sqrt{y})}{9}.$

23. $x^2 + y^2 = 18, x = \sqrt{3y}, x = 0, z = 0, z = \dfrac{10y}{11}.$

24. $x + y = 6, x = \sqrt{3y}, z = \dfrac{4x}{5}, z = 0.$

25. $y = \sqrt{15x}, y = \sqrt{15x}, z = 0, z = \sqrt{15}(1 + \sqrt{x}).$

26. $x^2 + y^2 = 50, y = \sqrt{5x}, y = 0, z = 0, z = \dfrac{3x}{11}.$

27. $x + y = 8, y = \sqrt{4x}, z = 3\,y, z = 0.$

28. $x = 16\sqrt{2y}, x = \sqrt{2y}, z = 0, z + y = 2.$

29. $x = 15\sqrt{y}, x = 15y, z = 0, z = 15(1 + \sqrt{y}).$

30. $x^2 + y^2 = 50, x = \sqrt{5y}, x = 0, z = 0, z = \dfrac{6y}{11}.$

31. $x = 17\sqrt{2y}, x = 2\sqrt{2y}, z = 0, z + y = \dfrac{1}{2}.$

Задача 11. Найти объем тела, заданного ограничивающими его поверхностями.

1. $x^2 + y^2 = 2y, z = \dfrac{5}{4} - x^2, z = 0.$

2. $x^2 + y^2 = y, x^2 + y^2 = 4y, z = \sqrt{x^2 + y^2}, z = 0.$

3. $x^2 + y^2 = 8\sqrt{2}x, z = x^2 + y^2 - 64, z = 0 (z \geq 0).$

4. $x^2 + y^2 + 4x = 0, z = 8 - y^2, z = 0.$

5. $x^2 + y^2 = 6x, x^2 + y^2 = 9x, z = \sqrt{x^2 + y^2}, z = 0, y = 0 (y \leq 0).$

6. $x^2 + y^2 = 6\sqrt{2}y, z = x^2 + y^2 - 36, z = 0 (z \geq 0)$.

7. $x^2 + y^2 = 2y, z = \dfrac{9}{4} - x^2, z = 0$.

8. $x^2 + y^2 = 2y, x^2 + y^2 = 5y, z = \sqrt{x^2 + y^2}, z = 0$.

9. $x^2 + y^2 + 2\sqrt{2}y = 0, z = x^2 + y^2 - 4, z = 0 (z \geq 0)$.

10. $x^2 + y^2 = 4x, z = 10 - y^2, z = 0$.

11. $x^2 + y^2 = 7x, x^2 + y^2 = 10x, z = \sqrt{x^2 + y^2}, z = 0, y = 0 (y \leq 0)$.

12. $x^2 + y^2 = 8\sqrt{2}y, z = x^2 + y^2 - 64, z = 0 (z \geq 0)$.

13. $x^2 + y^2 = 2y, z = \dfrac{13}{4} - x^2, z = 0$.

14. $x^2 + y^2 = 3y, x^2 + y^2 = 6y, z = \sqrt{x^2 + y^2}, z = 0$.

15. $x^2 + y^2 = 6\sqrt{2}x, z = x^2 + y^2 - 36, z = 0 (z \geq 0)$.

16. $x^2 + y^2 = 2\sqrt{2}y, z = x^2 + y^2 - 4, z = 0 (z \geq 0)$.

17. $x^2 + y^2 = 4x, z = 12 - y^2, z = 0$.

18. $x^2 + y^2 = 8x, x^2 + y^2 = 11x, z = \sqrt{x^2 + y^2}, z = 0, y = 0 (y \leq 0)$.

19. $x^2 + y^2 = 4\sqrt{2}x, z = \sqrt{x^2 + y^2} - 16, z = 0 (z \geq 0)$.

20. $x^2 + y^2 = 4y, z = 4 - x^2, z = 0$.

21. $x^2 + y^2 = 4y, x^2 + y^2 = 7y, z = \sqrt{x^2 + y^2}, z = 0$.

22. $x^2 + y^2 = 4\sqrt{2}y, z = x^2 + y^2 - 16, z = 0 (z \geq 0)$.

23. $x^2 + y^2 + 2x = 0, z = \dfrac{17}{4} - y^2, z = 0$.

24. $x^2 + y^2 = 9x, x^2 + y^2 = 12x, z = \sqrt{x^2 + y^2}, z = 0, y = 0 (y \geq 0)$.

25. $x^2 + y^2 + 2\sqrt{2}x = 0, z = x^2 + y^2 - 4, z = 0 (z \geq 0)$.

26. $x^2 + y^2 = 4y, z = 6 - x^2, z = 0$.

27. $x^2 + y^2 = 10x, x^2 + y^2 = 13x, z = \sqrt{x^2 + y^2}, z = 0, y = 0 (y \geq 0)$.

28. $x^2 + y^2 = 2\sqrt{2}x, z = x^2 + y^2 - 4, z = 0 (z \geq 0)$.

29. $x^2 + y^2 = 2x, z = \dfrac{21}{4} - y^2, z = 0$.

30. $x^2 + y^2 = 5y, x^2 + y^2 = 8y, z = \sqrt{x^2 + y^2}, z = 0$.

31. $x^2 + y^2 + 2x = 0, z = \dfrac{25}{4} - y^2, z = 0$.

Задача 12. Найти объем тела, заданного ограничивающими его поверхностями.

1. $y = 5x^2 + 2, y = 7, z = 3y^2 - 7x^2 - 2, z = 3y^2 - 7x^2 - 5$.

2. $y = 5x^2 - 2, y = -4x^2 + 7, z = 4 + 9x^2 + 5y^2, z = -1 + 9x^2 + 5y^2$.

3. $x = -5y^2 + 2, x = -3, z = 3x^2 + y^2 + 1, z = 3x^2 + y^2 - 5$.

4. $x = 2y^2 - 3, x = -7y^2 + 6, z = 1 + \sqrt{x^2 + 16y^2}, z = -3 + \sqrt{x^2 + 16y^2}$.

5. $y = -6x^2 + 8, y = 2, z = x - x^2 - y^2 - 1, z = x - x^2 - y^2 - 5$.

6. $y = 5x^2 - 1, y = -3x^2 + 1, z = -2 + \sqrt{3x^2 + y^2}, z = -5 + \sqrt{3x^2 + y^2}$.

7. $x = 5y^2 - 9, x = -4, z = x^2 + 4x - y^2 - 4, z = x^2 + 4x - y^2 + 2$.

8. $y = 6x^2 - 1, y = 5, z = 2x^2 + x - y^2, z = 2x^2 + x - y^2 + 4$.

9. $x = 5y^2 - 1, x = -3y^2 + 1, z = 2 - \sqrt{x^2 + 6y^2}, z = -1 + \sqrt{x^2 + 6y^2}$.

10. $x = -3y^2 + 7, x = 4, z = 2 + \sqrt{6x^2 + y^2}, z = 3 + \sqrt{6x^2 + y^2}$.

11. $y = -5x^2 + 3, y = -2, z = 2x^2 - 3y - 6y^2 - 1, z = 2x^2 - 3y - 6y^2 + 2$.

12. $y = x^2 - 5, y = -x^2 + 3, z = 4 + \sqrt{5x^2 + 8y^2}, z = 1 + \sqrt{5x^2 + 8y^2}$.

13. $x = 3y^2 - 5, x = -2, z = 2 - \sqrt{x^2 + 16y^2}, z = 8 - \sqrt{x^2 + 16y^2}$.

14. $x = y^2 - 2, x = -4y^2 + 3, z = \sqrt{16 - x^2 - y^2} + 2, z = \sqrt{16 - x^2 - y^2} - 1$.

15. $y = 2x^2 - 1, y = 1, z = x^2 - 5y^2 - 3, z = x^2 - 5y^2 - 6$.

16. $y = x^2 - 2, y = -4x^2 + 3, z = 2 + \sqrt{x^2 + y^2}, z = -1 + \sqrt{x^2 + y^2}$.

17. $x = -4y^2 + 1, x = -3, z = x^2 - 7y^2 - 1, z = x^2 - 7y^2 + 2$.

18. $x = 7y^2 - 6, x = -2y^2 + 3, z = 3 + 5x^2 - 8y^2, z = -2 + 5x^2 - 8y^2$.

19. $y = 1 - 2x^2, y = -1, z = x^2 + 2y + y^2 - 2, z = x^2 + 2y + y^2 + 1$.

20. $y = x^2 - 7, y = -8x^2 + 2, z = 3 - 12y^2 + 5x^2, z = -2 - 12y^2 + 5x^2$.

21. $x = 2y^2 + 3, x = 5, z = 1 + \sqrt{9x^2 + 4y^2}, z = 4 + \sqrt{9x^2 + 4y^2}$.

22. $y = 3x^2 + 4, y = 7, z = 5 - \sqrt{2x^2 + 3y^2}, z = 1 - \sqrt{2x^2 + 3y^2}$.

23. $x = 5y^2 - 2, x = -4y^2 + 7, z = 4 - \sqrt{2x^2 + 3y^2}, z = -1 - \sqrt{2x^2 + 3y^2}$.

24. $x = -2y^2 + 5, x = 3, z = 5 - \sqrt{x^2 + 25y^2}, z = 2 - \sqrt{x^2 + 25y^2}$.

25. $y = -3x^2 + 5, y = 2, x = 3 + \sqrt{5x^2 + y^2}, z = -1 + \sqrt{5x^2 + y^2}$.

26. $y = 3x^2 - 5, y = -6x^2 + 4, z = 2 + 10x^2 - y^2, z = -2 + 10x^2 - y^2$.

27. $x = 4y^2 + 2, x = 6, z = x^2 + 4y^2 + y + 1, z = x^2 + 4y^2 + y + 4$.

28. $x = 3y^2 - 2, x = -4y^2 + 5, z = 4 - 7x^2 - 9y^2, z = 1 - 7x^2 - 9y^2$.

29. $y = 2x^2 - 5, y = -3, z = 2 + \sqrt{x^2 + 4y^2}, z = -1 + \sqrt{x^2 + 4y^2}$.

30. $y = 2x^2 - 3, y = -7x^2 + 6, z = 1 - 5x^2 - 6y^2, z = -3 - 5x^2 - 6y^2$.

31. $y = -2x^2 + 7, y = 5, z = 1 - 2x^2 + 3y^2, z = 4 - 2x^2 + 3y^2$.

Задача 13. Найти объем тела, заданного ограничивающими его поверхностями.

1. $z = \sqrt{9 - x^2 - y^2}, \dfrac{9z}{2} = x^2 + y^2$.

2. $z = \dfrac{15\sqrt{x^2 + y^2}}{2}, z = \dfrac{17}{2} - x^2 - y^2$.

3. $z = \sqrt{4 - x^2 - y^2}, z = \sqrt{\dfrac{x^2 + y^2}{255}}$.

4. $z = \sqrt{64 - x^2 - y^2}, z = 1, x^2 + y^2 = 60$.
(внутри цилиндра)

5. $z = \sqrt{\dfrac{16}{9} - x^2 - y^2}, 2z = x^2 + y^2$.

6. $z = 3\sqrt{x^2 + y^2}, z = 10 - x^2 - y^2$.

7. $z = \sqrt{25 - x^2 - y^2}, z = \sqrt{\dfrac{x^2 + y^2}{99}}$.

8. $z = \sqrt{100 - x^2 - y^2}, z = 6, x^2 + y^2 = 51$.
(внутри цилиндра)

9. $z = \dfrac{21\sqrt{x^2 + y^2}}{2}, z = \dfrac{23}{2} - x^2 - y^2$.

10. $z = \sqrt{16 - x^2 - y^2}, 6z = x^2 + y^2$.

11. $z = \sqrt{9 - x^2 - y^2}, z = \sqrt{\dfrac{x^2 + y^2}{80}}$.

12. $z = \sqrt{81 - x^2 - y^2}, z = 5, x^2 + y^2 = 45$
(внутри цилиндра).

13. $z = \sqrt{1 - x^2 - y^2}, \dfrac{3z}{2} = x^2 + y^2$.

14. $z = 6\sqrt{x^2 + y^2}, z = 16 - x^2 - y^2$.

15. $z = \sqrt{36 - x^2 - y^2}, z = \sqrt{\dfrac{x^2 + y^2}{63}}$.

16. $z = \sqrt{64 - x^2 - y^2}, z = 4, x^2 + y^2 = 39$
(внутри цилиндра).

17. $z = \sqrt{144 - x^2 - y^2}, 18z = x^2 + y^2$.

18. $z = 3\dfrac{\sqrt{x^2 + y^2}}{2}, z = \dfrac{5}{2} - x^2 - y^2$.

19. $z = \sqrt{9 - x^2 - y^2}, z = \sqrt{\dfrac{x^2 + y^2}{35}}$.

20. $z = \sqrt{49 - x^2 - y^2}, z = 3, x^2 + y^2 = 33$
(внутри цилиндра).

21. $z = \sqrt{36 - x^2 - y^2}, 9z = x^2 + y^2$.

22. $z = 9\sqrt{x^2 + y^2}, z = 22 - x^2 - y^2$.

23. $z = \sqrt{16 - x^2 - y^2}, z = \sqrt{\dfrac{x^2 + y^2}{15}}$.

24. $z = \sqrt{36 - x^2 - y^2}, z = 2, x^2 + y^2 = 27$
(внутри цилиндра).

25. $z = \sqrt{\dfrac{4}{9} - x^2 - y^2}, z = x^2 + y^2$.

26. $z = 12\sqrt{x^2 + y^2}, z = 28 - x^2 - y^2$.

27. $z = \sqrt{9 - x^2 - y^2}, z = \sqrt{\dfrac{x^2 + y^2}{8}}$.

28. $z = \sqrt{25 - x^2 - y^2}, z = 1, x^2 + y^2 = 21$
(внутри цилиндра).

29. $z = \sqrt{64 - x^2 - y^2}, 12z = x^2 + y^2$.

30. $z = \dfrac{9\sqrt{x^2 + y^2}}{2}, z = \dfrac{11}{2} - x^2 - y^2$.

31. $z = \sqrt{36 - x^2 - y^2}, z = \sqrt{\dfrac{x^2 + y^2}{3}}$.

Задача 14. Найти объем тела, заданного ограничивающими его поверхностями.

1. $z = 2 - 12(x^2 + y^2), z = 24x + 2$.

2. $z = 10((x-1)^2 + y^2) + 1, z = 21 - 20x$.

3. $z = 8(x^2 + y^2) + 3, z = 16x + 3$.

4. $z = 2 - 20((x+1)^2 + y^2), z = -40x - 38$.

5. $z = 4 - 14(x^2 + y^2), z = 4 - 28x$.

6. $z = 28((x+1)^2 + y^2) + 3, z = 56x + 59$.

7. $z = 32(x^2 + y^2) + 3, z = 3 - 64x$.

8. $z = 4 - 6((x-1)^2 + y^2) + 1, z = 12x - 8$.

9. $z = 2 - 4(x^2 + y^2), z = 8x + 2$.

10. $z = 22((x-1)^2 + y^2) + 13z = 47 - 44x$.

11. $z = 24(x^2 + y^2) + 1, z = 48x + 1$.

12. $z = 2 - 18((x+1)^2 + y^2) + 1, z = -36x - 34$.

13. $z = -16(x^2 + y^2) - 1, z = -32x - 1$.

14. $z = 26((x+1)^2 + y^2) + 1, z = -52x - 2$.

15. $z = 2 - 12(x^2 + y^2), z = 24x + 2$.

16. $z = -2((x-1)^2 + y^2) - 1, z = 4x - 5$.

17. $z = -2(x^2 + y^2) - 1, z = 4y - 1$.

18. $z = 26((x-1)^2 + y^2) - 2, z = 50 - 52x$.

19. $z = 30(x^2 + y^2) + 1, z = 60y + 1.$

20. $z = -16((x+1)^2 + y^2) - 1, z = -32x - 33.$

21. $z = 2 - 18(x^2 + y^2), z = 2 - 36y.$

22. $z = 24((x+1)^2 + y^2) + 1, z = 48x + 49.$

23. $z = 22(x^2 + y^2) + 3, z = 3 - 44y.$

24. $z = 2 - 4((x-1)^2 + y^2), z = 8x - 6.$

25. $z = 4 - 6(x^2 + y^2), z = 12y + 4.$

26. $z = 32((x-1)^2 + y^2) + 3, z = 67 - 64x.$

27. $z = 28(x^2 + y^2) + 3, z = 56y + 3.$

28. $z = 4 - 14((x+1)^2 + y^2) + 1, z = -28x - 24.$

29. $z = 2 - 20(x^2 + y^2), z = 2 - 40y.$

30. $z = 8((x+1)^2 + y^2) + 3, z = 16x + 19.$

31. $z = 10(x^2 + y^2) + 1, z = 1 - 20y.$

Задачи 15. Найти объем тела, заданного неравенствами.

1. $1 \le x^2 + y^2 + z^2 \le 49, -\sqrt{\dfrac{x^2 + y^2}{35}} \le z \le \sqrt{\dfrac{x^2 + y^2}{3}}, -x \le y \le 0.$

2. $4 \le x^2 + y^2 + z^2 \le 649, \sqrt{\dfrac{x^2 + y^2}{15}} \le z \le \sqrt{\dfrac{x^2 + y^2}{3}}, -\sqrt{3}x \le y \le 0.$

3. $4 \le x^2 + y^2 + z^2 \le 64, z \ge \sqrt{\dfrac{x^2 + y^2}{3}}, -\dfrac{x}{\sqrt{3}} \le y \le 0.$

4. $4 \le x^2 + y^2 + z^2 \le 36, z \ge -\sqrt{\dfrac{x^2 + y^2}{63}}, 0 \le y \le \dfrac{x}{\sqrt{3}}.$

5. $1 \le x^2 + y^2 + z^2 \le 36, z \ge \sqrt{\dfrac{x^2 + y^2}{99}}, -\sqrt{3}x \le y \le \sqrt{3}x.$

6. $25 \le x^2 + y^2 + z^2 \le 100, z \le \sqrt{\dfrac{x^2 + y^2}{99}}, \sqrt{3}x \le y \le -\sqrt{3}x.$

7. $1 \le x^2 + y^2 + z^2 \le 49, 0 \le z \le \sqrt{\dfrac{x^2 + y^2}{24}}, -y \le -\dfrac{x}{\sqrt{3}}, y = -3x.$

8. $25 \le x^2 + y^2 + z^2 \le 121, -\sqrt{\dfrac{x^2 + y^2}{24}} \le z \le 0, y \ge -x\sqrt{3}, y \ge -\sqrt{3}x.$

9. $4 \le x^2 + y^2 + z^2 \le 64, -\sqrt{\dfrac{x^2 + y^2}{35}} \le z \le \sqrt{\dfrac{x^2 + y^2}{3}}, x \le y \le 0.$

10. $16 \le x^2 + y^2 + z^2 \le 100, -\sqrt{\dfrac{x^2 + y^2}{35}} \le z \le \sqrt{\dfrac{x^2 + y^2}{3}}, \sqrt{3}x \le y \le 0.$

11. $16 \le x^2 + y^2 + z^2 \le 100, z \le \sqrt{\dfrac{x^2 + y^2}{3}}, -\sqrt{3}x \le y \le -\dfrac{x}{\sqrt{3}}.$

12. $16 \le x^2 + y^2 + z^2 \le 64, z \ge -\sqrt{\dfrac{x^2 + y^2}{63}}, -\dfrac{x}{\sqrt{3}} \le y \le -\sqrt{3}x.$

13. $4 \le x^2 + y^2 + z^2 \le 49, z \ge \sqrt{\dfrac{x^2+y^2}{99}}, y \ge 0, y \le \sqrt{3}x.$

14. $36 \le x^2 + y^2 + z^2 \le 121, z \ge -\sqrt{\dfrac{x^2+y^2}{993}}, y \ge \sqrt{3}x, y \ge 0.$

15. $4 \le x^2 + y^2 + z^2 \le 64, 0 \le z \le \sqrt{\dfrac{x^2+y^2}{24}}, y \ge \sqrt{3}x, y \le \dfrac{x}{\sqrt{3}}.$

16. $36 \le x^2 + y^2 + z^2 \le 144, -\sqrt{\dfrac{x^2+y^2}{24}} \le z \le 0, y \ge \sqrt{3}x, y \ge \dfrac{x}{\sqrt{3}}.$

17. $9 \le x^2 + y^2 + z^2 \le 81, -\sqrt{\dfrac{x^2+y^2}{3}} \le z \le \sqrt{\dfrac{x^2+y^2}{35}}, 0 \le y \le -x.$

18. $36 \le x^2 + y^2 + z^2 \le 144, -\sqrt{\dfrac{x^2+y^2}{3}} \le z \le -\sqrt{\dfrac{x^2+y^2}{15}}, 0 \le y \le -\sqrt{3}x.$

19. $36 \le x^2 + y^2 + z^2 \le 144, z \le \sqrt{\dfrac{x^2+y^2}{3}}, \sqrt{3}x \le y \le \dfrac{x}{\sqrt{3}}.$

20. $36 \le x^2 + y^2 + z^2 \le 100, z \ge -\sqrt{\dfrac{x^2+y^2}{63}}, \dfrac{x}{\sqrt{3}} \le y \le \sqrt{3}x.$

21. $9 \le x^2 + y^2 + z^2 \le 64, z \ge \sqrt{\dfrac{x^2+y^2}{99}}, y \le \dfrac{x}{\sqrt{3}}, y \le -\dfrac{x}{\sqrt{3}}.$

22. $49 \le x^2 + y^2 + z^2 \le 144, z \le -\sqrt{\dfrac{x^2+y^2}{99}}, y \ge \dfrac{x}{\sqrt{3}}, y \ge -\dfrac{x}{\sqrt{3}}.$

23. $9 \le x^2 + y^2 + z^2 \le 81, 0 \le z \le \sqrt{\dfrac{x^2+y^2}{24}}, y \le 0, y \le \dfrac{x}{\sqrt{3}}.$

24. $49 \le x^2 + y^2 + z^2 \le 169, -\sqrt{\dfrac{x^2+y^2}{24}} \le z \le 0, y \ge 0, y \ge \dfrac{x}{\sqrt{3}}.$

25. $16 \le x^2 + y^2 + z^2 \le 100, -\sqrt{\dfrac{x^2+y^2}{3}} \le z \le \sqrt{\dfrac{x^2+y^2}{35}}, 0 \le y \le x.$

26. $64 \le x^2 + y^2 + z^2 \le 196, -\sqrt{\dfrac{x^2+y^2}{3}} \le z \le -\sqrt{\dfrac{x^2+y^2}{15}}, 0 \le y \le \sqrt{3}x.$

27. $64 \le x^2 + y^2 + z^2 \le 196, z \le \sqrt{\dfrac{x^2+y^2}{3}}, \dfrac{x}{\sqrt{3}} \le y \le 0.$

28. $64 \le x^2 + y^2 + z^2 \le 144, z \ge -\sqrt{\dfrac{x^2+y^2}{63}}, 0 \le y \le \dfrac{x}{\sqrt{3}}.$

29. $16 \le x^2 + y^2 + z^2 \le 81, z \ge \sqrt{\dfrac{x^2+y^2}{99}}, y \le 0, y \le -\sqrt{3}x.$

30. $64 \le x^2 + y^2 + z^2 \le 169, z \le -\sqrt{\dfrac{x^2+y^2}{99}}, y \ge 0, y \ge -\sqrt{3}x.$

31. $16 \le x^2 + y^2 + z^2 \le 100, 0 \le z \le \sqrt{\dfrac{x^2+y^2}{24}}, y \le 0, y \le -\dfrac{x}{\sqrt{3}}.$

Задача 16. Тело V задано ограничивающими его поверхностями, μ -плотность. Найти массу тела.

1. $64(x^2 + y^2) = z^2, x^2 + y^2 = 4, y = 0, z = 0 (y \ge 0, z \ge 0), \mu = \dfrac{5(x^2 + y^2)}{4}$.

2. $x^2 + y^2 + z^2 = 4, x^2 + y^2 = 1, (x^2 + y^2 \le 1), x = 0 (x \ge 0); \mu = 4|z|$.

3. $x^2 + y^2 = 1, x^2 + y^2 = 2z, x = 0, y = 0, z = 0 (x \ge 0, y \ge 0), \mu = 10x$.

4. $x^2 + y^2 = \dfrac{16}{49} z^2, x^2 + y^2 = \dfrac{4}{7} z, x = 0, y = 0 (x \ge 0, y \ge 0), \mu = 80yz$.

5. $x^2 + y^2 + z^2 = 1, x^2 + y^2 = 4z^2, x = 0, y = 0 (x \ge 0, y \ge 0, z \ge 0); \mu = 20z$.

6. $36(x^2 + y^2) = z^2, x^2 + y^2 = 1, x = 0, z = 0 (x \ge 0, z \ge 0), \mu = \dfrac{5}{6}(x^2 + y^2)$.

7. $x^2 + y^2 + z^2 = 16, x^2 + y^2 = 4 (x^2 + y^2 \le 4); \mu = 2|z|$.

8. $x^2 + y^2 = 4, x^2 + y^2 = 8z, x = 0, y = 0, z = 0 (x \ge 0, y \ge 0), \mu = 5x$.

9. $x^2 + y^2 = \dfrac{4}{25} z^2, x^2 + y^2 = \dfrac{2}{5} z, x = 0, y = 0 (x \ge 0, y \ge 0), \mu = 28xz$.

10. $x^2 + y^2 + z^2 = 4, x^2 + y^2 = z^2, x = 0, y = 0 (x \ge 0, y \ge 0, z \ge 0); \mu = 6z$.

11. $25(x^2 + y^2) = z^2, x^2 + y^2 = 4, x = 0, y = 0, z = 0 (x \ge 0, y \ge 0, z \ge 0), \mu = 2(x^2 + y^2)$.

12. $x^2 + y^2 + z^2 = 9, x^2 + y^2 = 4 (x^2 + y^2 \le 4), y = 0 (y \ge 0); \mu = |z|$.

13. $x^2 + y^2 = 1, x^2 + y^2 = 6z, x = 0, y = 0, z = 0 (x \ge 0, y \ge 0, z \ge 0), \mu = 90y$.

14. $x^2 + y^2 = \dfrac{z^2}{25}, x^2 + y^2 = \dfrac{z}{5}, x = 0, y = 0 (x \ge 0, y \ge 0), \mu = 14yz$.

15. $x^2 + y^2 + z^2 = 4, x^2 + y^2 = 9z^2, x = 0, y = 0 (x \ge 0, y \ge 0, z \ge 0); \mu = 10z$.

16. $9(x^2 + y^2) = z^2, x^2 + y^2 = 4, x = 0, y = 0, z = 0 (x \ge 0, y \ge 0, z \ge 0), \mu = \dfrac{5(x^2 + y^2)}{3}$.

17. $x^2 + y^2 + z^2 = 4, x^2 + y^2 = 1 (x^2 + y^2 \le 1), \mu = 6|z|$.

18. $x^2 + y^2 = 1, x^2 + y^2 = z, x = 0, y = 0, z = 0 (x \ge 0, y \ge 0), \mu = 10y$.

19. $x^2 + y^2 = \dfrac{z^2}{49}, x^2 + y^2 = \dfrac{z}{7}, x = 0, y = 0 (x \ge 0, y \ge 0), \mu = 10xz$.

20. $x^2 + y^2 + z^2 = 4, x^2 + y^2 = 4z^2, x = 0, y = 0 (x \ge 0, y \ge 0, z \ge 0), \mu = 10z$.

21. $16(x^2 + y^2) = z^2, x^2 + y^2 = 1, x = 0, y = 0, z = 0 (x \ge 0, y \ge 0, z \ge 0), \mu = 5(x^2 + y^2)$.

22. $x^2 + y^2 + z^2 = 16, x^2 + y^2 = 4 (x^2 + y^2 \le 4), \mu = |z|$.

23. $x^2 + y^2 = 4, x^2 + y^2 = 4z, x = 0, y = 0, z = 0 (x \ge 0, y \ge 0), \mu = 5y$.

24. $x^2 + y^2 = z^2, x^2 + y^2 = z, x = 0, y = 0 (x \ge 0, y \ge 0), \mu = 35yz$.

25. $x^2 + y^2 + z^2 = 1, x^2 + y^2 = z^2, x = 0, y = 0 (x \ge 0, y \ge 0, z \ge 0), \mu = 32z$.

26. $x^2 + y^2 = z^2, x^2 + y^2 = 4, x = 0, y = 0, z = 0 (x \ge 0, y \ge 0, z \ge 0), \mu = \dfrac{5(x^2 + y^2)}{2}$.

27. $x^2 + y^2 + z^2 = 9, x^2 + y^2 = 4 (x^2 + y^2 \le 4), z = 0 (z \ge 0), \mu = 2z$.

28. $x^2 + y^2 = 1, x^2 + y^2 = 3z, x = 0, y = 0, z = 0 (x \ge 0, y \ge 0, z \ge 0), \mu = 15x$.

29. $x^2 + y^2 = \dfrac{4z^2}{49}, x^2 + y^2 = \dfrac{2z}{7}, x = 0, y = 0 (x \ge 0, y \ge 0), \mu = 20xz$.

30. $x^2 + y^2 + z^2 = 16, x^2 + y^2 = 9z^2, x = 0, y = 0 (x \ge 0, y \ge 0, z \ge 0), \mu = 5z$.

31. $4(x^2 + y^2) = z^2, x^2 + y^2 = 1, y = 0, z = 0 (y \ge 0, z \ge 0), \mu = 10(x^2 + y^2)$.